About the Origin of the Lorentz Field Contraction

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When a charge is moving in vacuum at a constant velocity, its intrinsic electromagnetic field undergoes a Lorentz contraction. This fact is common knowledge. But it is not so for the direct physical cause for that contraction. If relativity itself is not used as such a cause[1], it is necessary to apply to classical electrodynamics, which fortunately has never been given up by relativity. The direct cause for the Lorentz contraction of the field becomes obvious as soon as a difference between two electromagnetic fields is examined of which one field belongs to the charge at rest while the other one refers to the same charge in motion. As will be shown below, this difference is generated according to the law of electromagnetic induction and may be called a contracting field as being responsible for the contraction and all its details. Since the divergence of this contracting field is zero, it represents a vortical (solenoidal) electric field, which is caused by the motion of the charge with a constant velocity and satisfies the free Maxwell equations whose right-hand parts are zero. In front of a moving positive charge, this field is directed mainly against the motion, while behind the charge its action is aligned with motion. Such field weakens the longitudinal intrinsic electric field both in front of the charge and behind it, which is nothing else but the Lorentz contraction of the resulting field. Having examined the properties of that field, we will be able to qualitatively explain the Lorentz contraction not only without relativity but even without any mathematics, so that the explanation becomes understandable even on a high school level.

1. Introduction

When one is asked about the physical reason responsible for the Lorentz contraction of an intrinsic electromagnetic field of an electric charge moving in vacuum with a constant velocity, a traditional response usually sounds like this: "The Lorentz contraction takes place, because otherwise the principle of relativity would be violated". In other words, the Lorentz contraction of the field (as well as many other relativistic effects) is usually presented as coming from the blue – without any concern about the physical details of its origination. Actually, every relativistic effect has its own good reasons to obey the principle of relativity [1] [2]. Such reasons, often connected with dynamic prehistory of the effect, are sometimes called Lorentz boosts [2]. They make the well-known relativistic effects more tangible and comprehensible when explained

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[1] Using relativity as a cause for a certain relativistic effect such as a Lorentz field contraction is the same as to explain the result of an elastic ball collision in terms of the laws of energy and momentum conservation – everything is correct but you miss all direct physical causes for what is going on, such as deformations of the elastic balls. This similarity between electrodynamics and mechanics is borrowed from Einstein's "Autobiographical Notes", where Einstein estimates his special relativity as not a theory but rather a restricting principle, such as the law of energy conservation [8].

in terms of electrodynamics rather than relativity. Moreover, sometimes there arises a situation where relativity proves helpless even as a formal tool for solving the problem. A good example was given by E. N. Parker [3]. Imagine a single charge which is moving through a continuous medium with its dielectric constant so high that the speed of light inside the medium is many times lower than the speed of light in vacuum. If the dielectric constant is sufficiently high, then such a charge is able to undergo the Lorentz contraction when moving at a speed of an ordinary car. Since the traditional Lorentz transformations are not applicable to this case and are to be replaced with their analog derived for a material medium, special relativity has nothing to do there and has to be replaced by classical electrodynamics just by technical reasons. Remaining valid as ever relativity must give the way to electrodynamics to receive a detailed physical description of the event and confirm that this description makes relativity only clearer and does not contradict to it in any way. All the more such electrodynamical consideration is applicable in vacuum with the properties of the contraction being very similar to those that take place in a material medium.

2. A direct causes for the Lorentz contraction of the electromagnetic field

To expose the direct causes for the Lorentz contraction of the electromagnetic field, it is sufficient to consider the difference between the electric field of a positive point charge, moving in a vacuum with a constant velocity, and the electric field of the same charge at rest. Of course, it does not matter what method is chosen to obtain these fields - whether they are derived by means of relativity [4] or obtained as a direct solution for Maxwell's equations [5]. In the laboratory frame of reference with cylindrical coordinates r, φ, z, (the z-axis is coincident with the trajectory of the charge and aligned with the motion), the components of the resulting field may be presented in the following traditional form [4]:

$$E_z = \frac{q\gamma Z}{(\gamma^2 Z^2 + r^2)^{3/2}}; \qquad E_r = \frac{q\gamma r}{(\gamma^2 Z^2 + r^2)^{3/2}}; \qquad E_\phi = 0; \qquad (1)$$

where $\gamma = 1/(1-\beta^2)^{1/2}$ is a Lorentz-factor, $\beta = \mathrm{v}/c$ is the velocity of the charge related to the speed of light, $Z = z - \mathrm{v}t$ is the distance between the reference point and the charge q, measured parallel to the trajectory of the charge in the laboratory frame, r is a distance between the reference point and the trajectory of the charge. Since this electric field is always directed along a position vector drawn between the charge and the reference point, it makes sense to change the frame of reference for a spherical one with coordinates R, φ, θ, where $R = (Z^2 + r^2)^{1/2}$ is the distance between the reference point and the charge, θ is an inclination of the position vector to the trajectory of the charge, so that $Z = R\cos\theta$ and $r = R\sin\theta$ .Then only one component of the electric field will be different from zero as was to be expected:

$$E_\varphi = 0; \quad E_\theta = 0; \qquad E_R = \frac{q}{\gamma^2 R^2 (1-\beta^2 \sin^2\theta)^{3/2}}; \qquad (2)$$

To obtain the desired difference between the two electric fields, the value of the potential field E_R corresponding to $\beta = 0$ must be subtracted from (2), which brings about the following result:

$$E_{\phi L} = E_{\theta \mathrm{L}} = 0; \qquad E_{RL} = \frac{q}{R^2}\left[\frac{1\text{-}\beta^2}{(1-\beta^2\sin^2\theta\,)^{3/2}} - 1\right]; \qquad (3)$$

where the first item in the brackets represents the total field of the charge with uniform velocity, while the second item ("-1") represents the field of the same charge, when the latter is at rest. The difference (3) might be called a contracting electric field $\mathbf{E_L}$, since it is responsible for the Lorentz contraction. It is always directed along a position vector which connects the reference point with the current position of the charge. Its sign, however, depends on the angle θ. For small values of θ it is always negative, i.e., E_L is directed toward the positive moving charge and suppresses the longitudinal potential field in front of the charge as well as behind it. As for the region of high latitudes θ, field $\mathbf{E_L}$ is positive there, i.e., it is directed away from the positive charge in the lateral directions and increases the transverse field.

The contracting electric field $\mathbf{E_L}$ is purely vortical, with its divergence equal to zero throughout the whole space, in contrast to the resulting field of the charge at rest, which is purely potential with its vorticity equal to zero at every point of space. As for the case of a charge with uniform velocity, its field might be considered as superposition of two fields: the vortical field $\mathbf{E_L}$ and the potential field, so that the resulting field of the moving charge has both divergence and vorticity differing from zero. The pattern of field $\mathbf{E_L}$ belonging to a moving relativistic particle $(\beta = 0.941)$ is shown in Fig.1. The motion takes place in a horizontal direction. The field of the particle consists of two electric fluxes – the inner flux whose lines of force are coming from infinity and converging upon the moving positive charge, and the outer flux whose lines of force are diverging from the charge to go away toward infinity. All lines of force are reflected from the charge and form loops which close at infinity. None of the lines terminates in the charge. The two electric fluxes, the inner and the outer, are separated from each other by a cone (shown in Fig. 1 with dashed lines), whose axis coincides with the trajectory of the charge and whose apex is at the charge itself. The half-angle θ_0 of this cone can be obtained from the expression (3) by setting this expression equal to zero:

$$\cos\theta_0 = \frac{(\gamma^{2/3} - 1)^{1/2}}{\beta\,\gamma}; \qquad (4)$$

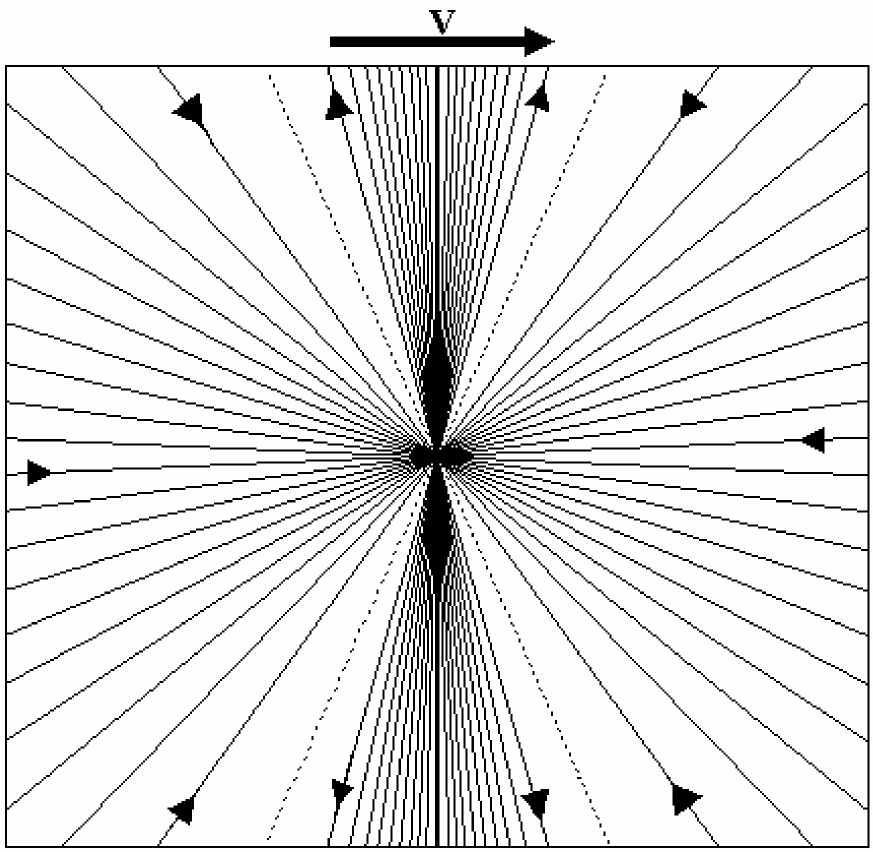

Fig. 1. The vortical part of an intrinsic electric field of a relativistic positive point charge *q* which moves with a constant velocity from left to right. This field is defined as a difference between the two intrinsic fields, which correspond to the point charge being respectively either in motion with a uniform velocity or at rest. $\beta = 0.941$. (This corresponds to a positron with energy *1 MeV).* The total flux of the vortical field makes *39.2 %* of the total flux $4\pi q$.

The angle θ_0 depends on the energy of the charge. When γ becomes infinitely large and β approaches 1, angle θ_0 is close to 90^0, as was to be expected. In this ultrarelativistic limit, the inner flux of the vortical field (3) completely counterbalances the potential field everywhere but the transverse plane of the moving charge. In this limit case, the configuration of the outer flux is a flat disk with infinite density, oriented perpendicularly to the trajectory of the charge. When energy decreases, so does θ_0, and it might be expected

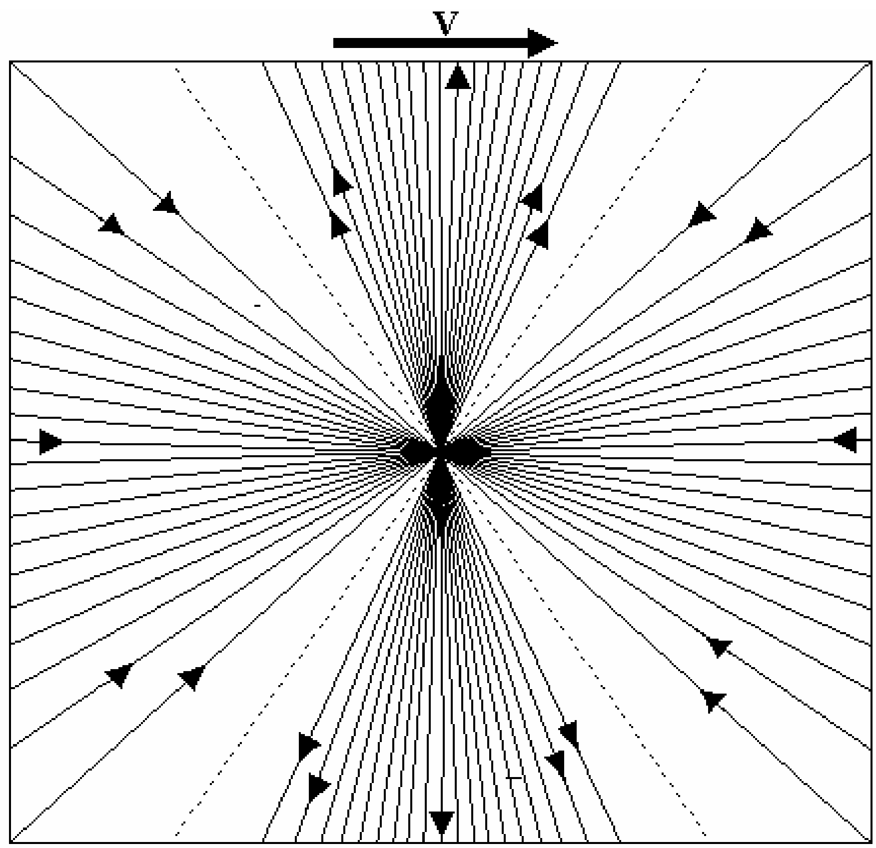

Fig. 2. The snapshot of the Lorentz field similar to that in Fig.1, but for a non-relativistic region $\beta = 0.14$. The total flux of the vortex field makes only 0.38 % of $4\pi q$.

that θ_0 should approach zero when β becomes infinitesimal. But in reality, its behavior turns out to be trickier. The separating cone never folds, no matter how low the speed of the charge might be. When passing to the non-relativistic limit, the induced electric flux vanishes, but the half-angle θ_0 of the separating cone never becomes less than $\cos^{-1}(1/3) = 54.7^0$, as follows from (4).

As for the value of the flux of the vortex field, it behaves without any tricks. With increasing the energy of the charge, the flux starts from zero for the charge at rest and ends in the value $4\pi q$, which corresponds to an infinite kinetic energy of the particle. To make sure of it, the field (3) must be integrated with respect to all current values of θ between 0 and θ_0 (or between θ_0 and $\pi/2$), which brings about the following formula:

$$\Phi = 4\pi\, q \frac{(\gamma^{2/3} - 1)^{3/2}}{\beta\,\gamma}; \qquad (5)$$

that defines the total value of the electric flux induced by the motion with constant velocity. This formula would have the same appearance as (4), if not for the power 3/2, which is substantially higher than in (4). That's why in the non-relativistic limit, the flux Φ is approaching zero, while the half-angle θ_0 of the separating cone remains finite. The pattern of the Lorentz field in the vicinity of this limit (at $\beta = 0.14$) is shown in Fig. 2. As calculations show, further decrease in β does not affect the field pattern in a noticeable way, because it takes place in a non-relativistic range of kinetic energy, where the flux of the contracting field is too weak to affect the pattern of the resulting field. As for the strength of this weak field, it is just proportional to β until it becomes non-linear at relativistic energies.

The difference in the pattern of the field between Figs. 1 and 2 can be seen visually. As for the value of the electric flux, it is so different for these two patterns that it would be impossible to retain the scale between the magnitude of the flux and the density of the lines of force for both the drawings. The lower the energy of the particle, the weaker the vortical field, as was to be expected.

3. The point of the issue given without any mathematics

The physical origin of the vortex field (3) illustrated by Figs. 1 and 2 can be explained qualitatively even without any mathematics. Let us imagine that there are a lot of magnetic probes around the charge's trajectory. Let all of them be fixed to the laboratory frame. What will they show? Since the magnetic field of a moving charge is distributed in space not evenly, (the farther from the charge, the weaker the field), every fixed probe will indicate that the magnetic field varies with time. In other words, the trajectory of the charge will be encircled with a changing magnetic flux. There cannot be any doubt that, according to the law of electromagnetic induction, this flux will generate a vortical electric field, whose flux in its turn will encircle the local fluxes of the magnetic field. According to Lenz's rule, this electric flux will try to oppose to all attempts of the current (whose role is played by the moving charge) to change its magnetic field. This means that in front of the positive charge the lengthwise component of the vortical electric field will be directed against the motion (the "current" tends to "appear" there), while behind the charge it will have the opposite direction (the "current" tends to "disappear" there). In other words, both before the charge and behind it, the vortical electric field suppresses the potential field, which would be created by the charge if it were at rest. Having converged on the charge both from the front and from behind, the lines of force of the vortical field cannot terminate in the charge and have to be reflected from the charge in lateral directions raising there the electrical flux. Eventually, this means contraction of the field in the direction of the motion. As for the exact pattern of the contracted field (whose lines of force remain straight lines in spite of changes in their density), it can be proven only by means of strict derivation, either based on relativity or obtained directly from Maxwell's equations. Of course, the first method is usually much simpler at least in the case of the charge moving through a vacuum.

4. Conclusion

The explanation of the Lorentz field contraction given above does not differ from that given by Lorentz [6], but it is not opposed to relativity in any way. On the contrary, it offers one more opportunity to demonstrate the Lorentz boosts approach before extending it to more complicated relativistic effects. As for Lorentz's ether, which may serve as a temporary substitute for the laboratory frame, it will automatically fade away once all other relativistic effects are taken into account [7]. This article was written in 1999 and updated in 2005. More details may be found in the book “Non-Postulated Relativity by the same author [9]. It is shown there that not only the Lorentz contraction but also all other relativistic effects have their own good reasons to take place. These reasons often do not catch the eye and some effort is needed to touch their roots.

5. References

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